M01

1) $\vec{a} = \dfrac{d\vec{v}}{dt} = \left| \begin{array}{l} 6t+1 \\ 0 \\ 3 \end{array} \right.$ 2) $\vec{OM} = \int \vec{v}\,dt = \left| \begin{array}{l} t^3+2 \\ 3 \\ \frac{3}{2}t^2+2t+2 \end{array} \right.$

M02

$x = 1+3t \Rightarrow t = \dfrac{x-1}{3} \Rightarrow y = 1+\dfrac{4}{3}(x-1)$ soit $y = \dfrac{4}{3}x - \dfrac{1}{3}$

$\vec{v} = \dfrac{d\vec{OM}}{dt} = \left| \begin{array}{l} 3 \\ 4 \end{array} \right.$ $\|\vec{v}\| = 5 \text{ m.s}^{-1}$ $\|\vec{a}\| = 0$

M03

1) $t = \dfrac{x}{2} \Rightarrow z = -5\left(\dfrac{x}{2}\right)^2 + 4\left(\dfrac{x}{2}\right)$ soit $\left\{ \begin{array}{l} z = -\frac{5}{4}x^2 + 2x \\ y = 0 \end{array} \right.$

2) * Sommet $\Rightarrow \dot{z}_S = 0$

$\dot{x} = 2 \text{ m.s}^{-1}$ donc $\vec{v}_S = 2\,\vec{i}$

* $z = 0 = (-5t+4)t \Rightarrow t = \dfrac{4}{5}$ s

$\dot{z} = -10t + 4 = -10\left(\dfrac{4}{5}\right) + 4 = -4 \text{ m.s}^{-1} \Rightarrow \vec{v}(z=0) = \left| \begin{array}{l} 2 \\ 0 \\ -4 \end{array} \right.$

3) $\vec{a} = \dfrac{d\vec{v}}{dt} = \left| \begin{array}{l} 0 \\ 0 \\ -10 \end{array} \right.$

M04

1) $t = -z \Rightarrow x = 1-z$ et $y = 1+2z$. La trajectoire est donc la droite intersection des 2 plans $\left\{ \begin{array}{l} x+z=1 \\ y-2z=1 \end{array} \right.$

2) $\cos t = \dfrac{x-\sqrt{3}}{2}$ et $\cos^2 t + \sin^2 t = 1$

cad $\left(\dfrac{x-\sqrt{3}}{2}\right)^2 + y^2 = 1$ ellipse dans le plan xOy

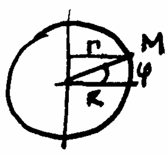
centrée en $x=\sqrt{3}$ $y=0$, de $\frac{1}{2}$ grand axe 2 et $\frac{1}{2}$ petit axe 1

M07

1) 1 jour sidéral = 23h 56 min = T

2) $\omega = \dfrac{2\pi}{T} = 7{,}292.10^{-5} \text{ rad.s}^{-1}$

3)



$r = R\cos\varphi$

$\|\vec{v}\| = \omega r = \omega R \cos\varphi$

$\|\vec{a}\| = \dfrac{v^2}{r} = \omega^2 r = \omega^2 R\cos\varphi$

4) $v = 461 \text{ m.s}^{-1}$

$a = 3{,}36.10^{-2} \text{ m.s}^{-2}$

$a \approx \dfrac{g}{1000}$

M08

Données et notations : $\alpha_0 = 0$, $\dot{\alpha}_0 = 1200$ tr. min^{-1} , $\ddot{\alpha}_0 = $ cste

$\alpha_f = 300$ tr $\quad \dot{\alpha}_f = 0$

1) $\dot{\alpha}(t) = \int \ddot{\alpha}_0 \, dt = \ddot{\alpha}_0 t + \dot{\alpha}_0$

$\alpha(t) = \int \dot{\alpha} \, dt = \int (\ddot{\alpha}_0 t + \dot{\alpha}_0) dt = \frac{1}{2} \ddot{\alpha}_0 t^2 + \dot{\alpha}_0 t$

2) $\ddot{\alpha}_0$? $\quad t_f$?

$$\begin{cases} \alpha_f = \frac{1}{2} \ddot{\alpha}_0 t_f^2 + \dot{\alpha}_0 t_f \\ \dot{\alpha}_f = 0 = \ddot{\alpha}_0 t_f + \dot{\alpha}_0 \Rightarrow t_f = -\frac{\dot{\alpha}_0}{\ddot{\alpha}_0} \end{cases}$$

$$\alpha_f = \frac{1}{2} \ddot{\alpha}_0 \frac{\dot{\alpha}_0^2}{\ddot{\alpha}_0^2} - \dot{\alpha}_0 \frac{\dot{\alpha}_0}{\ddot{\alpha}_0} = -\frac{1}{2} \frac{\dot{\alpha}_0^2}{\ddot{\alpha}_0}$$

d'où $\ddot{\alpha}_0 = -\frac{1}{2} \frac{\dot{\alpha}_0^2}{\alpha_f} = \boxed{-2400 \text{ tr. min}^{-1}}$

alors $\boxed{t_f = 0{,}5 \text{ min} = 30 \text{ s}}$

M09

$R_1 = 5$ cm , $\omega_0 = 180$ rad. s^{-1} , $R_2 = 30$ cm $\quad \omega_2$?

1) $v_1 = \omega_0 R_1 = v_2 = \omega_2 R_2 \Rightarrow \boxed{\omega_2 = \omega_0 \frac{R_1}{R_2}}$ A.N $\boxed{\omega_2 = 30 \text{ rad. s}^{-1}}$

2) a) C sur poulie 1

$$\vec{a}_1 = \frac{d\|\vec{v}\|}{dt} \vec{u}_T + \frac{v^2}{R_1} \vec{u}_N = \omega_0^2 R_1 \vec{u}_N \qquad \|\vec{v}\| \text{ cste}$$

$\boxed{a_1 = \omega_0^2 R_1}$ A.N. $\boxed{a_1 = 1620 \text{ m. s}^{-2}}$

b) C sur poulie 2 de même

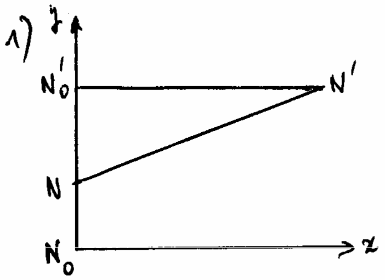
$\boxed{a_2 = \omega_2^2 R_2}$ A.N. $\boxed{a_2 = 270 \text{ m. s}^{-2}}$

c) C entre les 2 poulies

$\|\vec{v}\| = $ cste + mvt rectiligne $\Rightarrow \boxed{a_0 = 0}$

M10

1)



$\overrightarrow{N_0N} \left| \begin{array}{l} 0 \\ vt \end{array} \right.$ $\quad \overrightarrow{N_0N'} \left| \begin{array}{l} v't \\ a \end{array} \right.$

$\overrightarrow{NN'} \left| \begin{array}{l} v't \\ a - vt \end{array} \right.$

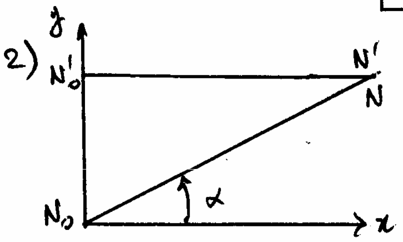
$D^2 = \|\overrightarrow{NN'}\|^2 = (v'^2 + v^2)t^2 - 2avt + a^2$

D min ? Etude de D^2 : $\dfrac{dD^2}{dt} = (v'^2 + v')2t - 2av$

d'où $t_{min} = \dfrac{av}{v'^2 + v^2}$ $\quad \left(\dfrac{dD^2}{dt} = 0\right)$ $\quad \dfrac{d^2(D^2)}{dt^2} > 0 \rightarrow$ c'est bien un minimum

alors Dmin = ... $\boxed{D_{min} = \dfrac{av'}{\sqrt{v^2 + v'^2}}}$

2)



$N_0N = vt$, $N_0N'_0 = a$, $N'_0N' = v't$

Pythagore $\Rightarrow$ $a^2 + v'^2t^2 = v^2t^2$

d'où $\boxed{t = \dfrac{a}{\sqrt{v^2 - v'^2}}}$ si $v' < v$, normal.

$\cos\alpha = \dfrac{N'_0N'}{N_0N} = \dfrac{v't}{vt}$ d'où $\boxed{\alpha = \arccos \dfrac{v'}{v}}$ si $v' < v$

M29

1) on étudie le système ponctuel "goutte" dans le réf. terrestre, supposé galiléen

bilan des forces : $\vec{P} = m\vec{g}$ $\quad \vec{f} = -\lambda\vec{v}$

RFDP $\quad m\vec{a} = \Sigma\vec{F} \Rightarrow \vec{a} = \vec{g} - \frac{\lambda}{m}\vec{v}$

projection $\left\{ \begin{array}{l} \ddot{x} = -\frac{\lambda}{m}\dot{x} \\ \ddot{y} = -\frac{\lambda}{m}\dot{y} \\ \ddot{z} = -g - \frac{\lambda}{m}\dot{z} \quad (\text{ou } \dot{v} = -g - \frac{\lambda}{m}v) \end{array} \right.$ $\quad$ (z vertical ascendant)

mvt selon $Oz \Rightarrow x$ et y cstes ?

cond. init. $\left\{ \begin{array}{lll} x = 0 & y = 0 & z = h \\ \dot{x} = 0 & \dot{y} = 0 & \dot{z} = 0 \end{array} \right.$

A $t = 0$, la seule force qui agit est selon Oz (le poids seul) $\Rightarrow$ $\dot{x}$ et $\dot{y}$ ne seront pas modifiés $\Rightarrow \forall t \quad x = y = 0$

$\Rightarrow$ <u>Le mvt a lieu selon Oz</u>

2) A partir de l'instant initial, $z \searrow$ et $|\dot{z}| \nearrow$ alors $\|\vec{f}\| \nearrow$ s'opposant au poids. Tant que $\|\vec{f}\| < \|\vec{P}\|$ la goutte accélère jusqu'à ce que $\|\vec{f}\| = \lambda\|\vec{v}\| = \|\vec{P}\|$ alors $v = cste = v_L$

$\dot{v} = -g - \frac{\lambda}{m}v \Rightarrow 0 = -g - \frac{\lambda}{m}v_L$ cad $v_L = -\frac{mg}{\lambda} < 0$

$\boxed{\|\vec{v}_L\| = \frac{mg}{\lambda}}$

3) $\dot{v} + \frac{\lambda}{m}v = -g$ $\quad$ SSSM : $\frac{dv}{dt} + \frac{\lambda}{m}v = 0$

$\Rightarrow \frac{dv}{v} = -\frac{\lambda}{m}dt \Rightarrow v = A\,e^{-\frac{\lambda}{m}t}$ $\quad$ (A cste)

SPASM : $v = -\frac{mg}{\lambda}$ $\quad (= v_L)$

SG : $v = -\frac{mg}{\lambda} + A\,e^{-\frac{\lambda}{m}t}$ $\quad$ A ? A $t = 0$ $\quad 0 = -\frac{mg}{\lambda} + A$

d'où $A = \frac{mg}{\lambda} \Rightarrow v = -\frac{mg}{\lambda}\left(1 - e^{-\frac{\lambda}{m}t}\right) \Rightarrow \boxed{\|\vec{v}\| = \frac{mg}{\lambda}\left(1 - e^{-\frac{\lambda}{m}t}\right)}$

4) $\frac{\|\vec{v}\|}{\|\vec{v}_L\|} = 1 - e^{-\frac{\lambda}{m}t_0} = 0{,}99$ $\quad$ t_0 = durée de la chute

$\Rightarrow \cdots \Rightarrow t_0 = \frac{\|\vec{v}_L\|}{g}\ln 100$ $\quad \boxed{t_0 = 2{,}35 \text{ ms}}$